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ADAPTIVE IMAGE ESTIMATION USING REDUCED UPDATE FILTERS,(U)
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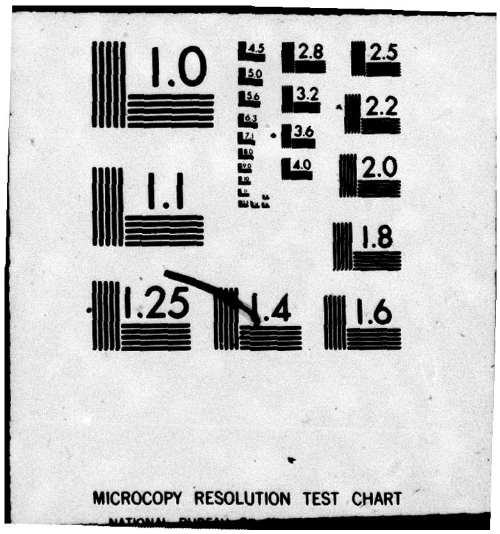
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ADAPTIVE IMAGE ESTIMATION USING REDUCED UPDATE FILTERS*

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ABSTRACT

When an image is estimated from noisy data using a linear shift-invariant (LSI) filter, the subjective improvement is relatively poor at low signal-to-noise ratios. This occurs for at least two reasons: first, the statistics of the image are markedly space-variant and second, the eye is very sensitive to blurring of edges. However adaptive filtering techniques can be applied to improve the subjective quality of noisy images even at low signal-to-noise ratios. This is accomplished in the present work by using multiple models to match the space-variant statistics and by using oriented edge models to prevent edge blurring in the filtered result.

INTRODUCTION

In the past several years various methods have been proposed for the recursive estimation of images and other two-dimensional (2-D) data. Most of these methods have been tied to first order or separable models that do not match image statistics very well. Also most of these methods are restricted to the homogeneous case where the model coefficients are constant.

Since many 2-D data fields, including images, are markedly non-homogeneous, there is a need for general recursive estimation procedures which can take this property into account. The 2-D Kalman filtering methods can theoretically take this into account, however the practical problem remains of obtaining the spatially varying model coefficients. Thus recent work has been concerned with the development of various adaptive estimators which will provide a practical means of estimating the model coefficients. A common property of these adaptive estimators is that they separate into two parts: one part estimates the model coefficients based directly on the noisy data and a second part filters the data using the estimated model coefficients.

Our original approach was based on the use of continuously updated model parameter estimates to readjust the Kalman gain (Ref. 1). However this approach was not very fruitful for the following reasons: Conventional one-dimensional (1-D) recursive least squares identification algorithms do not readily extend to 2-D recursive identification algorithms. Secondly, in order that the recursive identification algorithms be able to track space varying parameters, the effective bandwidth must be large enough that the variations are not smoothed over. This in turn makes the algorithm more susceptible to noise effects. Thirdly, the existence of process noise with an unknown v iance and measurement noise as well, can result in biased estimates if general least squares regression is performed directly on the measurements.

Presently it appears that the most favorable adaptive approach is a multiple model doubly stochastic procedure (Ref. 2) which combines a set of local Markov models of the field itself with a lower level 2-D Markov chain for the model parameter transitions. This development differs from previously published procedures in that it takes into account the need for transition probabilities between local models, rapid edge detection to prevent blurring, and near optimal local recursive estimation for general AR models.

2-D RECURSIVE ESTIMATION

Early attempts to achieve a truely recursive 2-D Kalman filter were of only limited success due to both the difficulty in establishing a suitable 2-D model and also the high dimension of the resulting state vector. In (Ref. 15) Woods and Radewan presented two new algorithms which, to a large extent, overcame the computational problems which had precluded the use of 2-D Kalman like processors. Both vector and scalar scanning methods were considered, but emphasis was placed on the scalar scan because it leads to processors which are recursive in both dimensions, i.e. 2-D recursive filters. These Kalman based algorithms allow the use of space-variant models which can provide a better match to local source statistics leading to a greater noise reduction with less signal distortion.

Other approaches to recursive image estimation include the work of Strintzis (Ref. 4), Jain (Ref. 5) and Murphy and Silverman (Ref. 6). In (Ref. 4), Strintzis presents an ARMA modeling approach to recursive processing coupled with an interesting additive rather than multiplicative based spectral decomposition. The addition of a moving average to the AR model of Kalman filtering can provide

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some modeling advantages, however in (Ref. 4) the developed model is for the correlation functions, not the field itself, thus simulation is not possible. Further only the steady state is considered, where the image filtering problem can be embedded in the class of cyclic processes. Unfortunately this embedding cannot be done if non-homogeneous image models are considered or if random boundary conditions are explicitly treated, as is done in (Ref. 7).

In (Ref. 5), Jain considers both implicit and explicit partial difference equation models for images. The models are compared and evaluated for mean-square improvement on representative images. This method is not suitable for space-variant or non-homogeneous image models. In (Ref. 6) Murphy and Silverman consider a vector scanning approach to image restoration. A constrained or reduced update vector processor is developed which nicely compliments the scalar reduced update filter of Woods and Radewan (Ref. 3). The problem of image restoration rather than simple noise filtering is considered in (Ref. 6). In (Ref. 7), Woods and Ingle extend the reduced update filter to the deconvolution case.

ADAPTIVE ESTIMATION

Development of a procedure for online parameter identification requires either a recursive type algorithm in which each estimate is a simply evaluated function of the previous estimate and the current measurement, or a non-recursive algorithm which can be executed in a time frame significantly less than a single pixel scanning period.

With regard to developing recursive algorithms suitable for use on 2-D fields, some type of approximation is desirable because of the nonlinear nature of simultaneously estimating both states and parameters. For example, a recursive least squares or weighted least squares algorithm (Refs. 8, 9) might be applied directly to the measurements in order to obtain parameter estimates which can then be used for Kalman gain adjustment. However, because these procedures give biased parameter estimates, Sen and Sinha (Ref. 10) have proposed a revised least squares procedure which results in approximately uncorrelated residuals. Alternate least squares type methods for reducing the bias include algorithms based on correlating the measurements (Ref. 11) and those based on correlating the estimates (Ref. 12).

An alternative to the above estimation based procedures which appears to be very powerful in view of recent results in adaptive control (Refs. 13, 14, 15), is based on the use of a bank of multiple models as per Lainiotis Partitioning Theorem (Ref. 16). Basically the Theorem says that under certain assumptions the nonlinear adaptive filter can be decomposed into two parts: a linear nonadaptive part consisting of a bank of Kalman filters, each conditioned on a predefined model, and a nonlinear adaptive part which evaluates the a posteriori model probabilities and forms the appropriate state estimate as a weighted sum of the filter outputs.

Below we summarize the development of a corresponding multiple model procedure for the estimation of images and other 2-D fields. For more details the reader is referred to (Refs. 2, 17, 18).

MULTIPLE MODEL RECURSIVE ESTIMATION

Following the notation in (Ref. 3), the dynamic model of the image formation is given as,

$$s(m,n) = \sum_{R_{\oplus+}} c_{ij}\, s(m-i,\, n-j) + w(m,n) \qquad (1)$$

where the coefficients are the model parameters, $R_{\oplus+}$ is the non-symmetric halfplane support of the model, and $w(m,n)$ is a white Gaussian noise having zero mean and variance σ_w^2. The scalar observation model is given by:

$$r(m,n) = s(m,n) + v(m,n) \qquad (2)$$

where, $v(m,n)$ is a white Gaussian noise with zero mean and known variance σ_v^2. Then using the global state vector $\underline{s}(m,n)$ as shown in Fig. 1, the vector equations for Eq. 1 and Eq. 2 can be written as:

$$\underline{s}(m,n) = \underline{C}\ \underline{s}(m-1,n) + \underline{w}(m,n) \qquad (3)$$

$$r(m,n) = \underline{h}^T \underline{s}(m,n) + v(m,n) \qquad (4)$$

where, $\underline{C}$ is the system propagation matrix determined by $\{c_{k\ell}\}$ and the ordering of the global state vector,

$$\underline{h} = [1, 0, 0, \ldots, 0]^T \qquad (5)$$

and

$$\underline{w}(m,n) = [v(m,n), 0, 0, \ldots, 0]^T . \qquad (6)$$

Let $\underline{s}_1(m,n)$ be the local state vector as shown in Fig. 1, and let $\underline{r}_1(m,n)$ and $\underline{v}_1(m,n)$ be the observation vector and white Gaussian noise vector respectively having the same support as that of the local state vector $\underline{s}_1(m,n)$. Note that the scalars $r(m,n)$ and $v(m,n)$ in eq. 2 are the leading elements of the vectors $\underline{r}_1(m,n)$ and $\underline{v}_1(m,n)$ respectively. Then from eq. 2,

$$\underline{r}_1(m,n) = \underline{s}_1(m,n) + \underline{v}_1(m,n) \qquad (7)$$

BASIC ADAPTIVE ALGORITHM

Now we make the following hypotheses to arrive at a basic multiple model description of an image:

1. At each pixel (m,n), there are L a-priori known classes $\{\theta_j\}_{j=1}^{L}$ of local state vectors distinguished by their direction of predominant correlation.

2. The probability distribution of the classes $\{p(\theta_j)\}_{j=1}^{L}$ is known a-priori.

3. The conditional distribution of the local state vector of a given class is Gaussian i.e.,

$$p\,(\underline{s}_1|\theta_j) \approx (2\pi)^{k/2}\,|\underline{R}_j|^{-1/2}\exp(-1/2\;\underline{s}_1^T\underline{R}_j\;\underline{s}_1) \tag{8}$$

where k = dimension of $\underline{s}_1$.

Hence in light of these hypotheses, the dynamical equation becomes

$$\underline{s}(m,n) = \underline{C}(\theta)\;\underline{s}(m-1,n) + \underline{w}(m,n) \tag{9}$$

where θ takes on one of L values at each (m,n).

Now a bank of reduced update Kalman filters running in parallel and each designed based upon the statistics of one of the L models can be designed (Ref. 3). Using the results developed in (Ref. 17), the decision logic is: Select model θ_j iff:

$$1/2\;\underline{r}_1^T(\underline{R}_j + \sigma_v^2\,\underline{I})^{-1}\underline{r}_1 + c_j \leq 1/2\;\underline{r}_1^T(\underline{R}_i + \sigma_v^2 I)^{-1}\underline{r}_1 + c_i, \quad i \neq j \tag{10}$$

$$\text{where}\quad c_j \triangleq 1/2\,\ell n|\underline{R}_j + \sigma_v^2\,\underline{I}| - \ell n P(\theta_j) \tag{11}$$

are constants which can be precomputed.

The above adaptive approach to the use of multiple models may be improved upon by introducing a spatial doubly stochastic Gaussian model as discussed next.

DOUBLY STOCHASTIC GAUSSIAN ESTIMATION

To introduce a doubly stochastic model, it is necessary to have a description for the underlying process which determines the elementary model to be used at each pixel. Such a description is the 2-D Markov chain which generates a discrete valued random field $\ell(m,n)\epsilon[\theta_j]$ specifying which of the L models is to be used at pixel (m,n).

The simplest example of sufficient generality would be a 2-D Markov chain with local state as shown in Fig. 2. Such a local state would require an L^4xL stochastic matrix to specify its transition probabilities. For the experimental cases to follow, L=5 was found to work rather well. In this case the transition matrix is 625x5. Each row of the transition matrix indicates the conditional probability of going from each of the L^4 possible present local states at (m-1,n) to the conditionally possible local states at (m,n). The 2-D Markov chain is thus specified by giving the support of the local state and its transition matrix $\underline{P}$.

Using the 2-D Markov chain to specify an underlying structure, one may generate a doubly stochastic model by using L different sets of parameters $\{c_{ij}^{\ell}\}$ to generate the random signal field. The resulting signal model would become

$$s(m,n) = \sum_{R_{\oplus+}} c_{ij}^{\ell(m,n)} s(m-i,n-j) + u(m,n) \tag{12}$$

where $\ell(m,n)$ is the 2-D Markov chain field with local state support consisting of a finite extent NSHP region similar to the local state support of Eq. 1. Thus given $\{\ell(m,n)\}$, the resulting signal model is space-variant Gaussian.

The filtering for such compound models can usefully be thought of as a two step process; at a given pixel one first estimates the local state of the underlying Markov chain, then one chooses the most likely model to do the estimation of the higher level random field. This will lead to a processor similar to the basic adaptive algorithm but with the added advantage that the local state of the underlying Markov chain can alter the likelihood of the various model transitions and hopefully improve upon model switching decisions.

Accordingly assume that s and ℓ are known for the past at pixel (m,n). The probability of $\ell(m,n)$, conditioned on the noisy observation of the local state $\underline{r}_1(m,n)$, can be written as

$$P[\ell(m,n)|\underline{r}_1(m,n),\text{ past }\ell] = \frac{p[\underline{r}_1(m,n)|\ell(m,n),\text{ past }\ell]P[\ell(m,n)|\underline{\ell}_1(m-1,n)]}{\sum_{\ell=1}^{L} p[\underline{r}_1(m,n)|\ell,\text{ past }\ell]P[\ell|\underline{\ell}_1(m-1,n)]} \tag{13}$$

where use has been made of the conditional probability of the 2-D Markov chain. The function $P(\cdot|\cdot)$ then is just given by the corresponding row of the probability transition matrix of the chain. The conditional probability density of $\underline{r}_1(m,n)$ is Gaussian with mean zero and covariance determined by the past and present chain data ℓ. If we assume all nearby points are in the same state ℓ, then we may approximate the conditional density of $\underline{r}_1$ as $N(\underline{0},\underline{R}_\ell+\sigma_v^2\,\underline{I})$. If we replace $\underline{\ell}_1(m-1,n)$ by its estimate $\hat{\underline{\ell}}_1(m-1,n)$, we obtain the following decision directed rule:

Choose ℓ such that

$$1/2\;\underline{r}_1^T(\underline{R}_\ell+\sigma_v^2\,\underline{I})^{-1}\underline{r}_1 + c_\ell(\hat{\underline{\ell}}_1(m-1,n)) = \min_{1\leq k\leq L} 1/2\;\underline{r}_1^T(\underline{R}_k + \sigma_v^2\,\underline{I})^{-1}\underline{r}_1 + c_k(\hat{\underline{\ell}}(m-1,n)) \tag{14}$$

where

$$c_\ell(\hat{\underline{\ell}}_1) \triangleq 1/2\;\ell n|\underline{R}_\ell+\sigma_v^2\,\underline{I}| - \ell n\;P[\ell\,|\,\hat{\underline{\ell}}_1(m-1,n)] \tag{15}$$

This decision rule can be compared with Eq. 10 and 11, the decision rule for basic algorithm. The difference is the replacement of the a-priori probabilities $P(\theta_i)$ by the conditional probabilities of the Markov chain given the decision-directed present state $\underline{\ell}_1(m-1,n)$. If the recent decisions have been good, this has the effect of improving the probability of the correct decision on the ℓ value at the present pixel.

RESULTS

The results of statistical measurements on sections of real images show that a good description of the images can be obtained with as few as five models. Four of the models correspond to the predominant correlation links in four directions making angles of 0°, 45°, 90° and 135° to the horizontal, for the images sampled and processed as rectangular arrays. The fifth model describes the parts of the image with isotropic structure. To test our algorithm we used a geometric image having the above mentioned four predominant edges and an isotropic structure.

This geometric image was a (128x128) density domain image, quantized to 8 bits (256 gray-levels), (Fig. 3a). For simulation purposes a signal-to-noise ratio of 3dB was set up with signal variance of unity and mean of zero (Fig. 3b). Second order steady state filters were designed for all models by using the reduced update error covariance equations. Fig. 3c shows the estimate obtained by the doubly stochastic Gaussian algorithm. The signal-to-noise ratio of Fig. 3c is 13.86 dB, thus the improvement with respect to the 3dB input image is I=10.86 dB.

We also processed a face image (Fig. 4a) at signal-to-noise ratio 3 dB. The noisy density domain image is shown in Fig. 4b. The five models were determined from the noise-free data of Fig. 4a using a mask determined by using the decision logic of Eq. 10 based on the geometrical image models. Thus this is the result of the first step in a possible iterative procedure which can tune the edge and isotropic models to a particular image. The result is seen in Fig. 4c which has a signal-to-noise ratio of 12.4 dB which is equivalent to an improvement I=9.4 dB.

Example comparisons with constant coefficient, linear estimate results are contained in references (2, 17, 18) and the reader is referred there to see those pictures.

CONCLUSIONS

The recently developed doubly-stochastic Gaussian estimator has been summarized and presented as a logical outgrowth of a basic adaptive algorithm based on the Partitioning Theorem and the 2-D reduced update filters.

Experimental results were presented using the new estimator for two test images. The subjective and numerical results demonstrate the utility of the adaptive approach.

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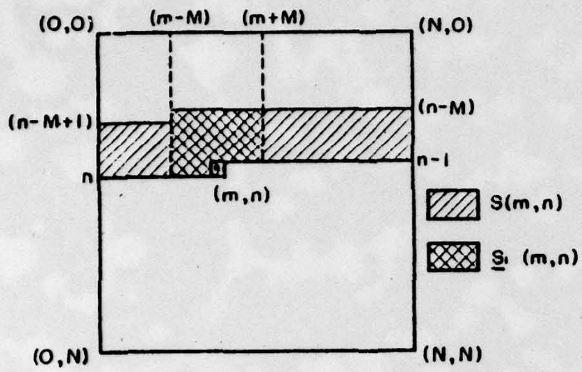
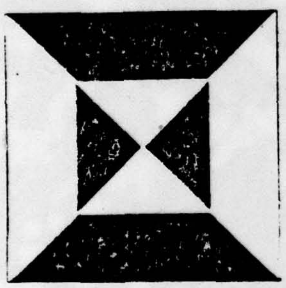


Fig. 1 Global and Local State Vector Support

Fig. 2 Local State of Markov Chain



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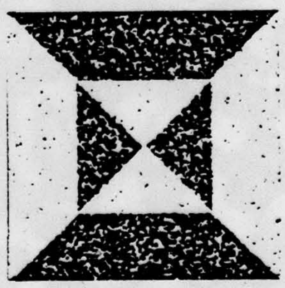
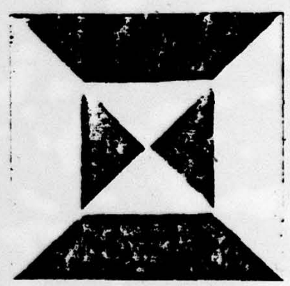


(b) noisy



(c) estimate

Fig. 3 Geometric Image

(a) original

(b) noisy

(c) estimate

Fig. 4 Face Image

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When an image is estimated from noisy data using a linear shift-invariant (LSI) filter, the subjective improvement is relatively poor at low signal-to-noise ratios. This occurs for at least two reasons: first, the statistics of the image are markedly space-variant and second, the eye is very sensitive to blurring of edges. However, adaptive filtering techniques can be applied to improve the subjective quality of noisy images even at low signal-to-noise ratios. This is accomplished in the present work by using multiple models to match the space-invariant statistics and by using oriented edge models to

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